NONLINEAR WAVES IN A VISCOELASTIC ROD AND THE PROBLEM OF IMPACT OF A FINITE ROD ON A RIGID OBSTACLE

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UDC 539.374.678:532

A number of general results in the dynamical theory of nonlinear elasticity has been obtained in [1] without taking account of dissipative phenomena, where a considerable quantity of interesting problems have also been solved. Relationships on strong and weak discontinuities in hereditary media of general type have been studied in [2]. However, making the form of the heredity functional specific is necessary for the solution of diverse dynamic problems. The papers [3, 4] are devoted to applying this problem to a description of the rheological behavior of unvulcanized rubber, melts, and concentrated solutions.

The rheological relationships obtained in [3, 4] are confirmed in a number of papers, and their good agreement with experimental data is shown (see [3] for some results of this confirmation and references).

A number of papers ([5-7], for example) are devoted to wave propagation in rods with different rheological characteristics, where slightly deformable rigid materials have been examined, in which connection the authors of these papers neglected the change in section in the usual rod approximation. The quadratic inertial forces were neglected here, and the center of gravity was carried over to an investigation of wave effects associated with the physical nonlinearity of the rheological equations.

Different types of waves being propagated in a viscoelastic rod are examined in this paper with strong geometric and physical nonlinearities taken into account.

Let us use the description averaged with respect to the section to investigate the isothermal motion of a viscoelastic rod by considering the motion of the rod as close to uniaxial tension-compression. It is assumed that all the quantities vary sufficiently slowly along the rod length, and the latter is much greater than the transverse dimension of the section.

The following equations are valid for an incompressible viscoelastic medium with nonlinear rheological equations of Maxwell type in this case:

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial x}(vf) = 0; \tag{1}$$

$$\frac{\partial (vf)}{\partial t} + \frac{\partial}{\partial x}\left(v^2 f - \sigma f \rho_0^{-1}\right) = 0; \tag{2}$$

$$\frac{1}{\lambda}\left(\frac{\partial \lambda}{\partial t} + v\frac{\partial \lambda}{\partial x}\right) + \frac{F(\lambda)}{6\theta_0} = \frac{\partial v}{\partial x}, \tag{3}$$

where f is the cross-sectional area of the rod; v is the mean velocity with respect to the section; $\sigma(\lambda)$ is the mean normal stress with respect to the section; λ is the mean elastic strain with respect to the section; x is the longitudinal coordinate; t is the time, $\theta_0 = \eta_0/2\mu$ is the relaxation time; η_0 is the viscosity under shear; and 2μ is Hooke's modulus. The rheological parameters θ_0, η_0, and μ govern the behavior of the medium in its linear strain domain. In the case of homogeneous uniaxial strain, the quantity λ is the ratio between the length of the specimen at a given time and its length after an instantaneous unloading. It hence follows that $\lambda > 1$ under tension and $0 < \lambda < 1$ under compression.

Equation (1) corresponds to mass conservation (2) to momentum conservation (we neglect gravity and surface tension forces), and (3) is the relaxation equation. The specifics of a Maxwell medium is to give the functions $\sigma(\lambda)$ and $F(\lambda)$. In the simplest case considered in [3]:

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 143-153, January-February, 1978. Original article submitted January 18, 1977.

 0021-8944/78/1901-0118$07.50 © 1978 Plenum Publishing Corporation

$$\sigma(\lambda) = 2\mu(\lambda^2 - \lambda^{-1}),$$
$$F(\lambda) = \lambda^{-2}(\lambda^2 - 1)(\lambda^2 + \lambda + 1)\exp\{-(1/2)\beta\lambda^{-2}(\lambda - 1)^2(\lambda^2 + 4\lambda + 1)\}. \quad (4)$$

The formula for the stress $\sigma(\lambda)$ in (4) corresponds to the classical kinetic theory of rubber under tension–compression of the specimen. The quantity $F(\lambda)/6\theta_0$ is the rate of irreversible strain in the medium under consideration. The exponential factor in the formula for $F(\lambda)$ describes the abrupt growth of the characteristic relaxation time with the growth of the elastic strains in the medium. The numerical parameter $\beta (0 < \beta \leq 1)$ characterizes the flexibility of the macromolecules for the viscoelastic polymer medium under consideration [3].

For the case of no stresses $\lambda = 1$ corresponds; while to inertia-free homogeneous strain: $v = \varkappa(t)(x - x_0)$, $\sigma = \sigma(t)$, $\lambda = \lambda(t)$, $f = f(t)$ [3]. Letting $\theta_0 \to \infty$ (or $\beta \to \infty$) in (3), we obtain the limit case of an elastic medium with finite strains. Using (1), we can also convert (3) into a divergent form

$$\partial\lambda^{-1}/\partial t + \partial(v\lambda^{-1})/\partial x = F(\lambda)/6\theta_0\lambda. \quad (5)$$

Now, let us briefly examine the properties of the system (1)-(3). Composing the characteristic equation for this system by the usual means, we find the characteristic roots α_k

$$\alpha_{1,2} = v \pm \lambda\sqrt{\rho_0^{-1} d(\sigma/\lambda)/d\lambda}, \quad \alpha_3 = v. \quad (6)$$

It follows from (6) that the system is hyperbolic if $d(\sigma/\lambda)/d\lambda > 0$, as holds for the value of $\sigma(\lambda)$ determined from (4), since $d(\sigma/\lambda)/d\lambda = 2\mu(1 + 2\lambda^{-3}) > 0$ in this case.

It is known [8] that the characteristic roots α_k agree with the velocities $x_*'(t)$ for weak discontinuity propagation for which quantities from (1)-(3) are continuous along the line $x_*(t)$ in the x, t plane but their derivatives have jumps. In this case, an equation interrelating the quantities $f_*(t)$, $v_*(t)$, $\lambda_*(t)$ on the line $x_*(t)$ can be derived:

$$\frac{d}{dt}(v_* f_*) - (2v_* - x_*')\frac{df_*}{dt} + \frac{\rho_0^{-1}}{v_* - x_*'}\left(\frac{d\sigma}{d\lambda}\right)_*\left\{\frac{d}{dt}(f_*\lambda_*) + \frac{f_*\lambda_*}{6\theta}F(\lambda_*)\right\} = 0, \quad (7)$$

where $x_*'(t) = \alpha(t)$, $f_*(t) = f[x_*(t), t]$, and the other quantities in (7) are defined analogously.

It is known [1] that normal discontinuities in the stress and strain rate fields are missing for an incompressible nonlinear elastic medium. An analogous circumstance holds both for an incompressible viscoelastic medium of general type [2] and, therefore, for a medium of Maxwell type with the rheological equations (4). Nevertheless, as is seen from the system (1)-(3), the existence of strong discontinuities is possible for average equations of the "rod" approximation being considered. In the neighborhood of these discontinuities the average description of the motion of the viscoelastic medium being considered is naturally false, strictly speaking, and it is here necessary to use an essentially nonuniform scheme for the computations. However, it is known that outside the zone of abrupt changes in the parameters the average description yields a small error, and the zone of abrupt changes on the order of the rod diameter in the given average description is replaced by a discontinuity. It hence appears that all the qualitative features of such behavior can be described sufficiently well even within the framework of the one-dimensional model under consideration with strong discontinuities taken into account.

Let $x_0(t)$ be the shock coordinate. Let us mark all the quantities in (1), (2), (5) by the subscript 1 if they are examined for $x = x_0(t)_{+0}$, and by subscript 2 if they are taken for $x = x_0(t)_0$. Then by using the customary procedure to obtain conditions on the shock ([8], for instance), we have the following relationships from (1), (2), and (5):

$$x_0'(f_2 - f_1) + f_1 v_1 - f_2 v_2 = 0,$$
$$x_0'(f_2 v_2 - f_1 v_1) + f_1 v_1^2 - f_2 v_2^2 - \rho_0^{-1}(f_1\sigma_1 - f_2\sigma_2) = 0, \quad (8)$$
$$x_0'(1/\lambda_2 - 1/\lambda_1) + v_1/\lambda_1 - v_2/\lambda_2 = 0,$$

where $\sigma_k = \sigma(\lambda_k)$ $(k = 1, 2)$. It follows from(8) that

$$f_1\lambda_1 = f_2\lambda_2; \quad (9)$$

$$(x_0' - v_1)^2 = \frac{\lambda_1^2}{\rho_0(\lambda_1 - \lambda_2)}\left[\frac{\sigma(\lambda_1)}{\lambda_1} - \frac{\sigma(\lambda_2)}{\lambda_2}\right],$$
$$(x_0' - v_2)^2 = \frac{\lambda_2^2}{\rho_0(\lambda_2 - \lambda_1)}\left[\frac{\sigma(\lambda_2)}{\lambda_2} - \frac{\sigma(\lambda_1)}{\lambda_1}\right]. \quad (10)$$

The relationship (9) shows that a purely elastic strain is realized on the shock. If it is assumed that all the quantities (marked with the subscript 1) are known ahead of the shock, then the relationships (9) and (10) interrelate the unknown quantities v_2, f_2, λ_2, $x_0(t)$; the closing equation is obtained from the solution of the problem with initial data entirely for the system (1), (2), and (4).

Now, let us examine several examples illustrating strong and weak discontinuities in a viscoelastic rod. Let the wave be propagated in the positive x direction, i.e., $x_\alpha'(t) > 0$. Let I denote the domain $x < x_\alpha(t)$, and II the domain $x > x_\alpha(t)$. Here $x_\alpha = x_*(t)$ in the case of a weak discontinuity and $x_\alpha = x_0(t)$ in the case of a strong discontinuity.

Let us examine loading waves being propagated along an unloaded rod.

1. Weak Tension–Compression Waves. Here $f_1 = f_0 = \text{const}$, $v_1 = 0$. $\lambda_1 = 1$ in domain I. Because of continuity these same values are conserved on the weak discontinuity $x = x_*(t)$. Since $F(1) \equiv 0$, (7) is satisfied identically on the weak discontinuity. For $\lambda = 1$ it follows from (4) that $d(\sigma/\lambda)/d\lambda|_{\lambda=1} = 6\mu = E$, where E is the Young's modulus. We then have from (6) that $x_*' = (E/\rho_0)^{1/2} = c_0$; i.e., weak loading waves are propagated at a velocity c_0 of linear perturbation propagation (speed of sound) over an unloaded material.

2. Tension–Compression Shocks. In domain I as before, $f_1 = f_0 = \text{const}$, $v_1 = 0$, $\lambda_1 = 1$. Behind the shock, we have, from (9), (10), and (4),

$$f_2 = f_0/\lambda_2(t), \quad x_0' = c_0\sqrt{(1/3)(1 + \lambda_2^{-1} + \lambda_2^{-2})},$$
$$v_2 = x_0'(1 - \lambda_2), \quad c_0 = \sqrt{\frac{E}{\rho_0}}, \quad (11)$$

from which it follows that for a compression shock ($\lambda_2 < 1$) we have $f_2 > f_0$, $v_2 > 0$, $x_0' > c_0$; while for the tension shock ($\lambda_2 > 1$) we have $f_2 < f_0$, $v_2 < 0$, $x_0' < c_0$. This last inequality shows that the tension shock being propagated over an unloaded material is unstable since it will give energy to the small vibrations of the medium overtaking it during propagation. The compression shock is stable since it is propagated at supersonic speed and the small perturbations superposed on it will damp out because of radiation of the sound lagging behind it.

It hence follows that for sufficiently smooth initial conditions the compression wave front will be steeper disclosing the tendency for the strong discontinuity to appear; on the other hand, an initially sharp tension wave front will spread and become a weak discontinuity.

Now, let us examine unloading waves being propagated over a uniformly loaded relaxing rod.

3. Weak Unloading Waves. Here $f_1 = f_0 = \text{const}$, $\lambda_1 = \lambda_1(t)$, $v_1 = 0$ in domain I. It follows from (3) [or (7)] that $\lambda_1(t)$, $\sigma_1(t)$ are determined from the relationships

$$\frac{d\lambda_1}{dt} + \frac{\lambda_1}{6\theta_0}F(\lambda_1) = 0, \quad \sigma_1(t) = 2\mu(\lambda_1^2 - \lambda_1^{-1}). \quad (12)$$

These same values are also conserved on the weak discontinuity $x = x_*(t)$ because of continuity. It follows from (6) that the propagation velocity for a weak discontinuity is described by the formula

$$x_*'(t) = c_0\sqrt{(1/3)(\lambda_1^2 + 2\lambda_1^{-1})}. \quad (13)$$

Since $f(\lambda) = \lambda^2 + 2\lambda^{-1}$ has a minimum at $\lambda = 1$ and $f(1) = 3$, it follows from (13) that $x_*'(t) > c_0$, where there results from (12) that $x_*'(t) \to c_0$ as t grows. Therefore, weak unloading waves of the type under consideration are supersonic in a relaxing material for both tension and compression. Let us also note that relaxation of the material under inhomogeneous conditions occurs behind the wave front in domain II.

4. Unloading Shocks. In domain I as before $f_1 = f_*$, $\lambda_1 = \lambda_1(t)$, $v_1 = 0$, $\sigma_1 = \sigma(\lambda_1)$, where σ_1 and λ_1 are determined from (12). Behind the shock front, i.e., in domain II, the system (1)-(3) admits of the simple solution

$$\lambda = 1,\ v = v(t),\ \sigma \equiv 0, \tag{14}$$

while the equation

$$\partial f/\partial t + v(t)\partial f/\partial x = 0,$$

which is easily integrated, holds for f(x, t).

Taking account of (4) and (14), we have from (9) and (10) [$\lambda_2 = 1$, $v_2 = v(t)$]

$$f_2 = f_1\lambda_1,\quad x_0' = c_0\sqrt{(1/3)(\lambda_1^2 + \lambda_1 + 1},\quad v(t) = (1 - \lambda_1^{-1})x_0'. \tag{15}$$

It follows from (15) that for an unloading shock being propagated over a compressed material ($\lambda_1 < 1$), we have $f_2 < f_1$, $x_0' < c_0$, $v(t) < 0$ (i.e., this wave is unstable) while for an unloading shock being propagated over a material in tension ($\lambda_1 > 1$), we have $f_2 > f_1$, $x_0' > c_0$, $v(t) > 0$ (i.e., this wave is stable). In the case under consideration, a strong discontinuity will evidently be realized with the lapse of time for any initial data for an unloading wave being propagated over a rod in tension since $x_0'(t) > x_*'(t)$, while for an unloading wave being propagated over a compressed rod there will be a weak discontinuity since in this case $x_0'(t) < c_0 < x_*'(t)$.

Let us examine formulation of the problem about impact of a finite viscoelastic rod on a rigid obstacle. Let a rod of length L in the undeformed state and with a velocity U prior to impact make impact on a rigid obstacle.

Let us select the origin (x = 0) at the free end of the rod at the time it touches the wall. Then for the system (1)-(3) with (4) we shall have the following system of initial and boundary conditions:

$$\begin{aligned} &t = 0:\ v = U,\ f = f_0 = \text{const},\ \lambda = 1\ (0 \leqslant x < L);\\ &t > 0:\ v|_{x=L} = 0;\ \lambda|_{x=a(t)} = 1, \end{aligned} \tag{16}$$

where $a(t)$ is the coordinate of the free end of the rod. The boundary condition $\lambda = 1$ at $x = a(t)$ in (16) corresponds [see (4)] to the dynamical condition $\sigma = 0$ at the free end of the rod.

As is known, under the impact of a rod on a rigid obstacle, compression, unloading, and tension waves, which are successively replaced, are propagated. Hence, molecular adhesion forces originate at the point of contact from the instant of rod contact with the wall, which cause the rod to adhere to the wall. At the time t_* of origination of the first tension wave, the stress $\sigma(L, t_{*+0})$ in the rod at the point of its contact with the wall can exceed the stress σ_0 produced by the adhesion forces, and the rod separates from the wall with a mean mass velocity U_1 less than the initial velocity U because of the incompletely elastic nature of the impact. In the opposite case when $\sigma(L, t_{*+0}) < \sigma_0$. the rod adheres to the wall and damped vibrations originate for $t > t_*$. To estimate the quantity σ_0, it can be assumed in a rough approximation that $\sigma_0 = kE$, where $k \sim 1$ ($E = 6\mu$).

Resolution of the question of whether the rod separates from the wall upon impact or adheres to it depends not only on the magnitude of the initial rod velocity U, but also on the velocity at which the relaxation processes proceed which lower the stress level in the rod with the lapse of time.

Now, in addition to conditions (16), let us formulate the condition for separation or adhesion of the rod and the wall

$$\begin{aligned} &\sigma(L,\ t_{*+0}) > kE,\ v(L,\ t_{*+0}) < 0,\\ &\sigma(L,\ t_{*+0}) < kE,\ v(L,\ t_{*+0}) = 0\ (v_{x=L} = 0,\ t > t_*). \end{aligned} \tag{17}$$

The first condition in (17) corresponds to separation of the rod from the wall for $t > t_*$, and the second to the adhesion condition.

The formulas for the mean-mass velocity U_1 and the magnitude of the kinetic energy loss ΔE of the rod being separated from the wall are

$$U_1 = \frac{1}{f_0 L} \int_{a_*}^{L} v_*(x) f_*(x)\, dx \quad (f_0 L = V),$$
$$\Delta E = -(1/2)\rho_0 \int_{a_*}^{L} v_*^2(x) f_*(x)\, dx + (1/2)\rho_0 f_0 L v^2, \tag{18}$$

where V is the rod volume and $v_* = v|_{t_*+0}$, $f_* = f|_{t_*+0}$ are the velocity distribution and a section along the rod length at the time of its separation from the wall.

The solution of the problem formulated above about the impact of a viscoelastic rod on an obstacle is carried out by a numerical method in the dimensionless variables marked with a prime:

$$t = (2L/U)t',\ x = 2Lx',\ v = Uv',\ f = f_0 f',\ \sigma = 2\mu\sigma'. \tag{19}$$

Henceforth, the primes will be omitted for simplicity.

The following numerical values of the parameters

$$L = 50\ \text{cм},\ U = 10\text{—}10^3\ \text{cm/sec}.\ \theta_0 = 0.167\text{sec}, \rho_0 = 1\ \text{g/cm}^{-3},$$
$$\mu = 10^4\ \text{dyn/cm}^2$$

were taken in the computations.

After the passage to the dimensionless variables (19), the following parameters appear in these equations:

$$M_0 = Uc_0^{-1}, \quad \theta = 3\theta_0 U L^{-1} \quad (c_0 = (E\rho_0^{-1})^{1/2}),$$

where M_0 is the initial Mach number [the quantity M_0^{-2} appears in the dimensionless equation (2) as a factor for σ], and c_0 is the propagation velocity for linear perturbations. Therefore, the desired solution depends on three dimensionless parameters M_0, θ, β, where β enters into $F(\lambda)$ according to (4). For the mentioned numerical parameters $\theta = 2.4\ M_0$.

A symmetric formulation, under which the "wall" has the coordinate x = 0.5, was used for a numerical solution of the problem. In such a formulation the dimensionless initial and boundary conditions (17) take the form

$$t = 0:\ 0 \leqslant x \leqslant 1,\ f = \lambda = 1,\ v = \text{sgn}(x - 0.5);$$
$$t > 0:\ \lambda|_{x=a(t)} = \lambda|_{x=b(t)} = 1,$$

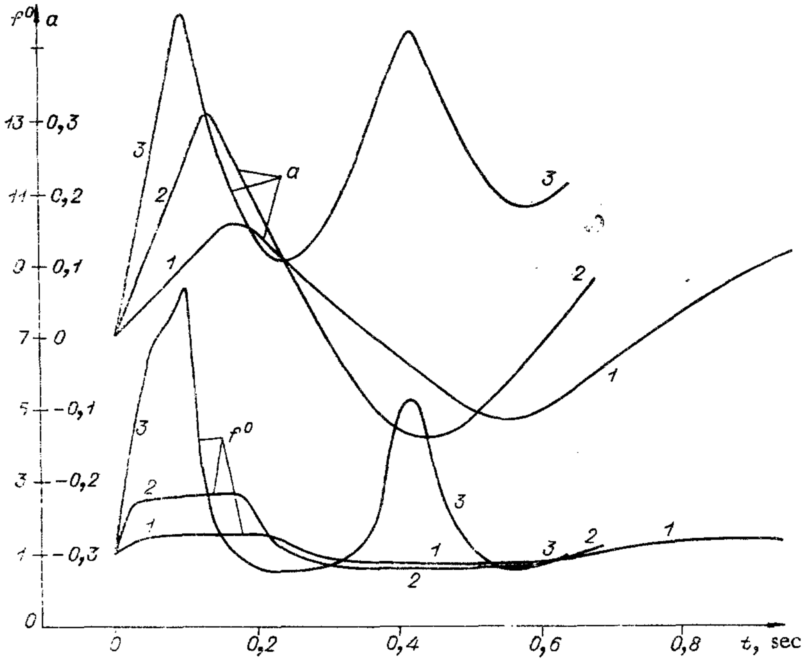
where $x = a(t)$, $x = b(t)$ are the coordinates of the free ends of the symmetrically continued rod at the time t. Use of the mentioned symmetry in the formulation of the problem got rid of the insertion of special approximations to calculate the unknown values of f, λ, and σ on the wall.

The problem was solved numerically by the method of a "through" computation of shocks, as well as the unloading and loading waves originating in the rod after impact. A divergent form of the continuity and momentum equations was used. The equation for the function $\lambda(x, t)$ was written in the following divergent form:

$$\frac{\partial A}{\partial t} + \frac{\partial B}{\partial x} + \frac{f^2 v}{\partial \theta_0} F(\lambda) = 0,$$
$$A(f, v, \lambda) = 2f \ln \lambda + 2f(\ln f - 1) + fv, \tag{20}$$
$$B(f, v, \lambda) = 2v \ln \lambda - (3M_0^2)^{-1}\sigma(\lambda) + v^2 + 2v(\ln f - 1),$$

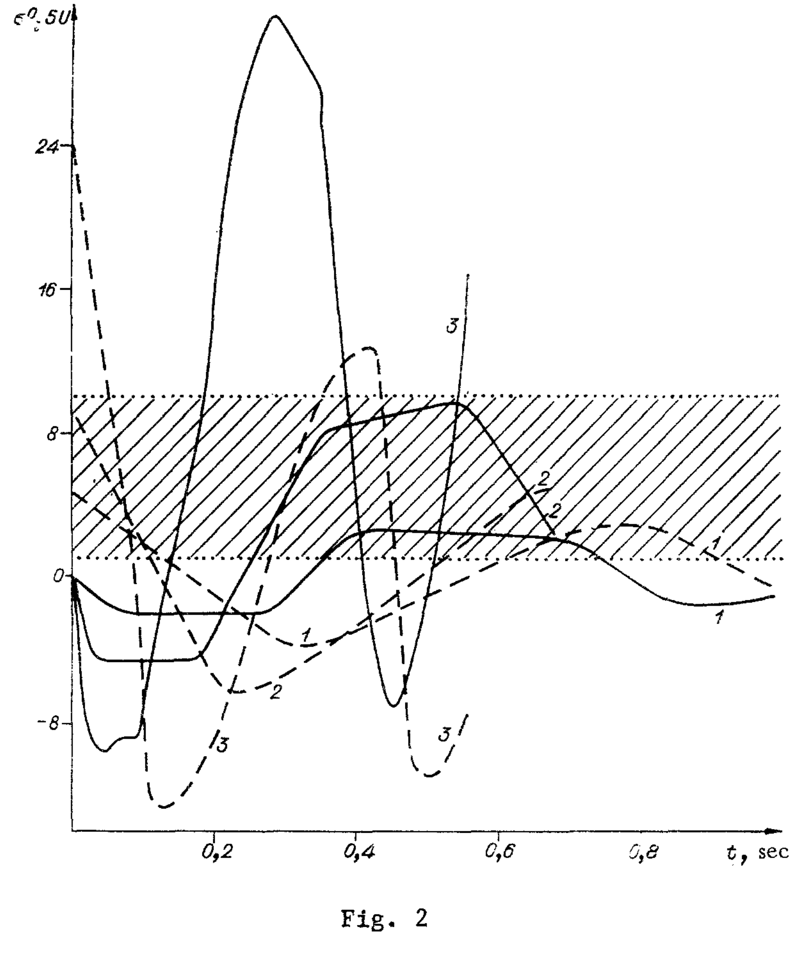
where $\sigma(\lambda)$, $F(\lambda)$ are dimensionless quantities determined from (4).

On the basis of a difference approximation of (20), first $\ln(\lambda)$ was determined, as is convenient for large strain zones. The computations were performed in the ξ coordinate system, where $\xi = [x - a(t)]/(b - a)$ related to the moving ends of the rod. Values of the functions f and v on the boundaries $\xi = 0$, $\xi = 1$ were determined on the basis of one-sided difference approximations for the continuity and momentum equations. The laws for motion of the free rod ends were computed by means of the values found for the velocities.



f^0 a
13 0,3
11 0,2
9 0,1
7 0
5 -0,1
3 -0,2
1 -0,3
0
a
f^0
0 0,2 0,4 0,6 0,8 t, sec

Fig. 1



ϵ^0, 5U
24
16
8
0
-8
0,2 0,4 0,6 0,8 t, sec

Fig. 2

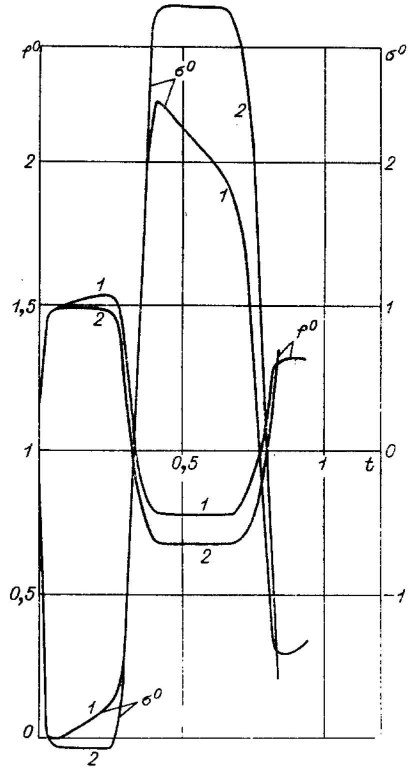


Fig. 3

The difference scheme is constructed by analogy to the known "Chekhard" scheme ([9], for instance) under the following modification. The emergence into the half-integer time layer was carried out on the basis of equations without "sources"("sinks"), due to the incomplete divergence of the approximating equations. The difference between the approximation and the "Chekhard" scheme is on the order of τ for smooth solutions.

The error in calculation was checked by the magnitude of the integrated imbalances in the mass and momentum as well as by comparing the results as the number of computational nodes increased. The results of the computations discussed below are obtained on a mesh with 80 nodes in the domain $0 \leq \xi \leq 1$.

Motions of a viscoelastic rod during its impact on a rigid obstacle were computed by the method described above. The initial Mach number varied between 0.4 and 2 for the relationship $\theta = 2.4M_0$ noted, which corresponded to initial rod flight velocities U between 100 and 500 cm/sec. Some results of the computations are represented in Figs. 1-4.

Time changes in the coordinate $x = a(t)$ of the free end of the rod and of the cross-sectional area $f^0(t) = f|_{x=1/2}$ of the rod at the point of its contact with the wall are shown in Fig. 1. The dimensional time [see (19)] is plotted along the horizontal. Curves 1-3 refer to the values $M_0 = 0.4$, 1, and 2 respectively, for the case $\beta = 0$ [see (4)]; it is seen that larger amplitudes and velocities of the damped wave processes with the more intense nonlinear distortions correspond to higher values of M_0.

It is interesting to note that, as is seen from Fig. 1, for $M_0 = 2$ the rod does not achieve the initial length during vibrations (for instance, the rod length diminishes approximately 20% at the end of the unloading phase) while the length grows compared to the initial length after the unloading phase for $M_0 = 0.4$ and 1. This is related to the fact that the velocities of the relaxation processes grow more rapidly with the growth of M_0 for $\beta = 0$ than do the wave propagation velocities. An analogous phenomenon can also hold for $\beta > 0$ in some range of M_0 variation.

The mean-mass rod velocities U_1 (dashed lines), calculated by means of (18), and the stresses σ^0 on the wall (solid lines) are represented in Fig. 2 as a function of the dimensional time t. Here, values of the stress according to the recoil criterion (17) with the value k = 0.35-3.5, which permits determination of the mean-mass rod velocity at the time of recoil from the wall, are denoted by the shaded strip. Curves 1-3 correspond to the same

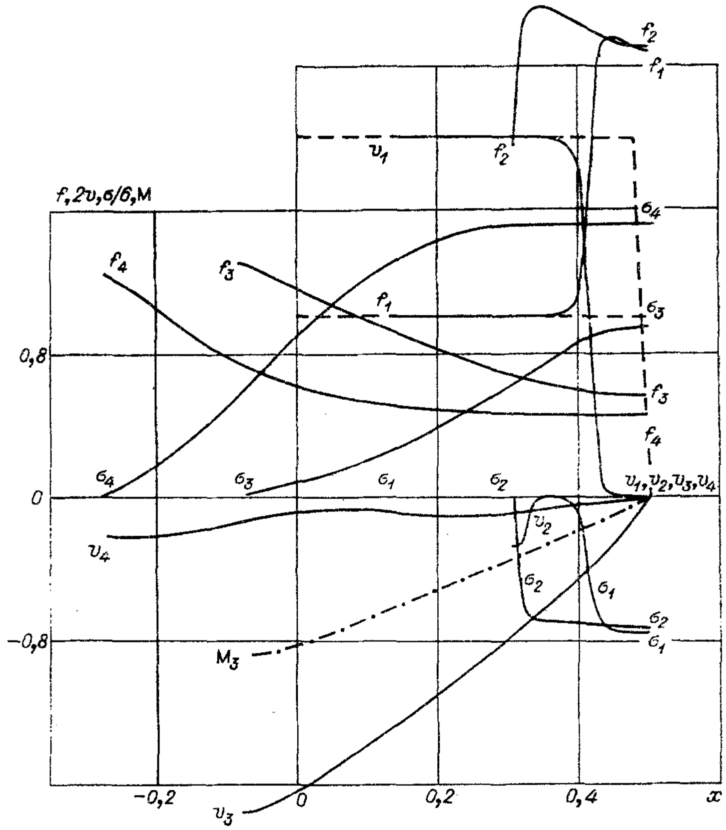


Fig. 4

values of M_0 as for Fig. 1, $\beta = 1$. Exactly as for the data of Fig. 1, larger amplitudes and frequencies of the dynamical variables σ^0 and U_1 correspond to larger values of M_0. As is seen from Fig. 2, larger rod recoil velocities from the obstacle hence correspond to larger values of M_0. The influence of the parameter β on certain quantities characterizing the wave process was moreover studied. Computations were performed for $M_0 = 0.4$ and $\beta = 0$, 1, and 40. Results of the computations for $\beta = 0$ and 1 differed insignificantly. The dependences $f^0(t)$ and $\sigma^0(t)$ for $M = 0.4$ are presented in Fig. 3 where the curves 1, 2 correspond to the values $\beta = 0$ and 40.

In this latter case, there are practically no relaxation processes in a viscoelastic rod except for the zone where $\lambda = 1$, and the rod behaves as an elastic body with large deformations, similar to cross-linked rubber.

It is seen from Fig. 3 that in contrast to $\beta = 40$, where there are quite definite plateaus in the dependences $\sigma^0(t)$, at $\beta = 0$ the stress amplitudes are less and there are sections with a noticeable change in $\sigma^0(t)$ instead of plateaus, which indicates intensive relaxation processes.

Distributions of values of the problem parameters along the rod for $M_0 = 1$, $\beta = 1$, are shown in Fig. 4 for different times. The dimensionless distributions of the quantities f, $\sigma/6$, 2U at the times $t_1 = 0.075$, $t_2 = 0.182$, $t_3 = 0.439$, $t_4 = 0.658$ sec are denoted by the subscripts 1-4. Distributions of the unperturbed quantities for $0 \leq x \leq 0.5$ are marked by a prime, where x = 0.5 corresponds to the free end of the rod. There is a distribution of parameters close to the shock being propagated from right to left at the time t_1 in a rod with a slightly smoothed initial velocity profile. At this time a considerable part of the rod has still not been subjected to perturbations (see the v_1, f_1, σ_1 distributions). Values calculated on the shock front agree well with the relationship (9) and (10) found. As is seen from the curve for f_1, relaxation occurs behind the shock front. This is seen more clearly from the curve f_2 for the time t_2 for which the shock has just been reflected from the free end of the rod (see curves 2 in Figs. 1 and 2 also). At this time almost the whole mass of the rod is at rest, except a small section around the free end, which has already started to move in the opposite direction (from right to left) at a low velocity. The σ_2 distribution also changes abruptly only near the free end. This time t_2 approximately corresponds to the starting-up loading phase of the compressed rod. In this case a shock cannot exist and a wave with a very spread out front is formed. This results in a sufficiently prolonged rebuilding of the section of the free rod end, whereupon relaxation processes succeed in occurring and the rod section at the free end is not completely restored after reflection of the wave. A tension phase starts after the unloading phase, as is illustrated in Fig. 4 by distributions of the quantities at the time t_3 (the initial tension stage) and at the time t_4 (the final tension stage). It is interesting to compare the mentioned times with the distributions 2 in Figs. 1 and 2.

A distribution of the local Mach numbers M at the time t_3, where $M^2 = M_0^2[3v^2/(\lambda^2 + 2\lambda^{-1})]$, which is similar to the v_3 distribution, is shown by the dash-dot line in Fig. 4.

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